

## STEP II, 2010, Q2

2 Prove that

$$\cos 3x = 4 \cos^3 x - 3 \cos x.$$

Find and prove a similar result for  $\sin 3x$  in terms of  $\sin x$ .

(i) Let

$$I(\alpha) = \int_0^\alpha (7 \sin x - 8 \sin^3 x) dx.$$

Show that

$$I(\alpha) = -\frac{8}{3}c^3 + c + \frac{5}{3},$$

where  $c = \cos \alpha$ . Write down one value of  $c$  for which  $I(\alpha) = 0$ .

(ii) Useless Eustace believes that

$$\int \sin^n x dx = \frac{\sin^{n+1} x}{n+1}$$

for  $n = 1, 2, 3, \dots$ . Show that Eustace would obtain the correct value of  $I(\beta)$ , where  $\cos \beta = -\frac{1}{6}$ .

Find all values of  $\alpha$  for which he would obtain the correct value of  $I(\alpha)$ .



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