# STEP II, 2010, Q13

**13** Rosalind wants to join the Stepney Chess Club. In order to be accepted, she must play a challenge match consisting of several games against Pardeep (the Club champion) and Quentin (the Club secretary), in which she must win at least one game against each of Pardeep and Quentin. From past experience, she knows that the probability of her winning a single game against Pardeep is $p$ and the probability of her winning a single game against Quentin is $q$, where $0 < p < q < 1$.

**(i)** The challenge match consists of three games. Before the match begins, Rosalind must choose either to play Pardeep twice and Quentin once or to play Quentin twice and Pardeep once. Show that she should choose to play Pardeep twice.

**(ii)** In order to ease the entry requirements, it is decided instead that the challenge match will consist of four games. Now, before the match begins, Rosalind must choose whether to play Pardeep three times and Quentin once (strategy 1), or to play Pardeep twice and Quentin twice (strategy 2) or to play Pardeep once and Quentin three times (strategy 3).

Show that, if $q - p > \frac{1}{2}$, Rosalind should choose strategy 1.

If $q - p < \frac{1}{2}$ give examples of values of $p$ and $q$ to show that strategy 2 can be better or worse than strategy 1.