STEP II, 2010, Q11

11 A uniform rod AB of length $4L$ and weight W is inclined at an angle θ to the horizontal. Its lower end A rests on a fixed support and the rod is held in equilibrium by a string attached to the rod at a point C which is $3L$ from A. The reaction of the support on the rod acts in a direction α to AC and the string is inclined at an angle β to CA. Show that

$$\cot\alpha = 3\tan\theta + 2\cot\beta\,.$$

Given that $\theta = 30°$ and $\beta = 45°$, show that $\alpha = 15°$.