# STEP II, 2009, Q9 MS

**9** (i) If you "break the lamina up" into a rectangle and a triangle (shapes whose geometric centres should be well-known to you), with relative masses 2 and 1, and impose (mentally, at least) a coordinate system onto the diagram, then the $x$-coordinate of the centre of mass is given by

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i} = \frac{2\times\frac{9}{2}+1\times 12}{3} = 7.$$

(ii) A more detailed approach, but still along similar lines, might be constructed in the following, tabular way:

| Shape | Mass | Dist. c.o.m. from $OZ$ |
|---|---|---|
| LH end | $540\rho$ | 7 |
| RH end | $540\rho$ | 7 |
| Front | $41d\rho$ | $\frac{27}{2}$ |
| Back | $40d\rho$ | 0 |
| Base | $9d\rho$ | $\frac{9}{2}$ |

Note that each mass has been calculated as area × density ($\rho$)

Then $x_E = \dfrac{2\times(540\rho)\times 7+41d\rho\times\frac{27}{2}+0+9dp\times\frac{9}{2}}{1080\rho+90d\rho}$, which (after much cancelling) simplifies to

$$= \frac{2\times 60\times 7+66d}{10(12+d)} = \frac{3(140+11d)}{5(12+d)}.$$

A similar approach for the full tank gives

| Object | Mass | Dist. c.o.m. from $OZ$ |
|---|---|---|
| Tank | $2880\rho$ | $\frac{27}{4}$ |
| Water | $10800k\rho$ | 7 |

and $x_F = \dfrac{2880\rho\times\frac{27}{4}+10800kp\times 7}{2880\rho+10800k\rho} = \dfrac{27+105k}{4+15k}$.