STEP II, 2009, Q8 MS

8 For the diagram, you are simply required to show P on AB, strictly between A and B; and Q on AC on other side of A to C. The two given parameters indicate that $CQ = \mu\, AC$ and $BP = \lambda\, AB$. Substituting these into the given expression, $CQ \times BP = AB \times AC \Rightarrow \mu AC\,.\,\lambda AB = AB\,.\,AC$ $\Rightarrow \mu = \frac{1}{\lambda}$. [Notice that CQ, BP, etc., are scalar quantities, and hence the "×" **cannot** be the vector product!]

Writing the equation of line PQ in the form $\mathbf{r} = t\,\mathbf{p} + (1-t)\,\mathbf{q}$ for some scalar parameter t and substituting the given forms for $\mathbf{p}$ and $\mathbf{q}$ gives $\mathbf{r} = t\lambda\,\mathbf{a} + t(1-\lambda)\mathbf{b} + (1-t)\mu\,\mathbf{a} + (1-t)(1-\mu)\mathbf{c}$. Eliminating $\mu = \frac{1}{\lambda} \Rightarrow \mathbf{r} = \left(t\lambda + \frac{1}{\lambda} - \frac{t}{\lambda}\right)\mathbf{a} + t(1-\lambda)\mathbf{b} + (1-t)\left(\frac{\lambda-1}{\lambda}\right)\mathbf{c}$. Comparing this to the given answer, we note that when $t = \frac{1}{1-\lambda}$ from the $\mathbf{b}$-component, $1 - t = \frac{\lambda}{\lambda-1}$, etc., so that we do indeed get $\mathbf{r} = -\mathbf{a} + \mathbf{b} + \mathbf{c}$, as required.

Since $\mathbf{d} - \mathbf{c} = \mathbf{b} - \mathbf{a}$, one pair of sides of opposite sides of $ABDC$ are equal and parallel, so we can conclude that $ABDC$ is a parallelogram