

## STEP II, 2009, Q8

- 8 The non-collinear points  $A$ ,  $B$  and  $C$  have position vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ , respectively. The points  $P$  and  $Q$  have position vectors  $\mathbf{p}$  and  $\mathbf{q}$ , respectively, given by

$$\mathbf{p} = \lambda\mathbf{a} + (1 - \lambda)\mathbf{b} \quad \text{and} \quad \mathbf{q} = \mu\mathbf{a} + (1 - \mu)\mathbf{c}$$

where  $0 < \lambda < 1$  and  $\mu > 1$ . Draw a diagram showing  $A$ ,  $B$ ,  $C$ ,  $P$  and  $Q$ .

Given that  $CQ \times BP = AB \times AC$ , find  $\mu$  in terms of  $\lambda$ , and show that, for all values of  $\lambda$ , the line  $PQ$  passes through the fixed point  $D$ , with position vector  $\mathbf{d}$  given by  $\mathbf{d} = -\mathbf{a} + \mathbf{b} + \mathbf{c}$ . What can be said about the quadrilateral  $ABDC$ ?



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