STEP II, 2009, Q7 EC

Q7 In many ways, this question is little more than an academic exercise, since I can see no way in which these integrals would actually arise in any practical situation. I apologise for this. However, it *was* a good test of candidates' ability to stretch a general result in different directions, probing them for increasing amounts of insight and perseverance. For future STEP-takers, the opening result is a good one for which to find a generalisation, and it is a possibly fruitful avenue to explore the "product rule of differentiation" for three terms (etc.), say $y = pqr$ in this case. Such an approach might have helped to prevent some of the ghastly mix-ups in writing all the terms out that were to be found in the scripts. It was disappointing to see that so few candidates seemed to think that they should tidy up the answer and demonstrate that the left-over bits did indeed form a cubic polynomial, as required by the question. In the end, we gave anyone the mark who simply observed that what was left was a cubic (if indeed that was the case in their working). Thereafter, (i) is a straightforward application of the result, requiring candidates only to identify the values of n, a and b. However, even here, it was rare to see folks justifying the form of the cubic, which might have acted as a check for errors. In (ii), the polynomial term is no longer cubic, so candidates were expected to try to see if an extra factor of $(x - 1)$ could be taken out to go with the other twenty-one $(x - 1)$s, which indeed it could. Checking the cubic's terms was rather more important here. The final integral, in (iii), *was* difficult, and this was where candidates were 'found out' on this question. The obvious thing is to try and extract some $(x - 2)$ factor(s) from the quartic polynomial, but this doesn't work. Candidates may reflect that they shouldn't have found this too much of a surprise, as that would simply have been repeating the "trick" of (ii). Though only a small minority realised it, the next most obvious possibility to try, having already found in (ii) that 'the next case up' gives a quartic rather than a cubic polynomial, is to try some combination of the obvious answer and the next one up, and this turns out to be exactly what is required.