STEP II, 2009, Q7

7 Let $y = (x-a)^n \mathrm{e}^{bx}\sqrt{1+x^2}$, where n and a are constants and b is a non-zero constant. Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x-a)^{n-1}\mathrm{e}^{bx}\mathrm{q}(x)}{\sqrt{1+x^2}},$$

where $\mathrm{q}(x)$ is a cubic polynomial.

Using this result, determine:

(i) $\displaystyle\int \frac{(x-4)^{14}\mathrm{e}^{4x}(4x^3-1)}{\sqrt{1+x^2}}\,\mathrm{d}x$;

(ii) $\displaystyle\int \frac{(x-1)^{21}\mathrm{e}^{12x}(12x^4-x^2-11)}{\sqrt{1+x^2}}\,\mathrm{d}x$;

(iii) $\displaystyle\int \frac{(x-2)^{6}\mathrm{e}^{4x}(4x^4+x^3-2)}{\sqrt{1+x^2}}\,\mathrm{d}x$.