STEP II, 2009, Q6 EC

Q6 This was another very popular question, but the one with the lowest mean mark score of all the pure questions, at about 7. I think that the initial enthusiasm of seeing something familiar in the *Fibonacci Numbers* was more than countered by the inequalities work that formed the bulk of the question. Nonetheless, I suspect that, if given the opportunity to talk it through after the event, many candidates would admit that half of the marks on the question are actually ludicrously easy to acquire and that they were really only put-off by appearances. For instance, to show that $S >$ any suitable lower-bound, one need only keep adding terms until the appropriate figure is exceeded. For those reciprocals of integers that are not easily calculated, such as $\frac{1}{13}$, it is perfectly reasonable to note something that they are greater than and us that in its place. Thus,

$$S = 1 + 1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{5} + \tfrac{1}{8} + \ldots > 1 + 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{5} + \tfrac{1}{10} + \ldots = 2 + 0.5 + 0.25 + 0.2 + 0.1 = 3.05 > 3$$

works pretty easily (though it may not have scored full marks in (i) as a particular approach was requested), and something similar could be made to work in showing that $S > 3.2$ in (ii). The approach that the question was designed to direct candidates towards was that of stopping the direct calculation at some suitable stage, and using an inequality of the form $\dfrac{1}{F_i} < \frac{1}{2}\left(\dfrac{1}{F_{i-2}}\right)$, possibly alternating with the odd- and even-numbered terms, to make the remaining sum less than a summable infinite GP. For further thoughts and possible developments of these ideas, I would refer the reader to the *Hints & Solutions* document for this paper.