

STEP II, 2009, Q4 MS

-
- 4(i) Writing $p(x) - 1 = q(x) \cdot (x - 1)^3$, where $q(x)$ is a quartic polynomial, immediately gives $p(1) = 1$.
- (ii) Diff⁸. using the product and chain rules leads to

$$p'(x) = q(x) \cdot 3(x - 1)^2 + q'(x) \cdot (x - 1)^3 = (x - 1)^2 \{3q(x) + (x - 1)q'(x)\},$$
 so that $p'(x)$ is divisible by $(x - 1)^2$.
- (iii) Similarly, we have that $p'(x)$ is divisible by $(x + 1)^2$ and $p(-1) = -1$.
 Thus $p'(x)$ is divisible by $(x + 1)^2 \cdot (x - 1)^2 = (x^2 - 1)^2$. However, $p'(x)$ is a polynomial of degree eight, hence $p'(x) = k(x^2 - 1)^4$ for some constant k . That is, $p'(x) = k(x^8 - 4x^6 + 6x^4 - 4x^2 + 1)$.
 Integrating term by term then gives $p(x) = k\left(\frac{1}{9}x^9 - \frac{4}{7}x^7 + \frac{6}{5}x^5 - \frac{4}{3}x^3 + x\right) + C$, and use of both $p(1) = 1$ and $p(-1) = -1$ help to find k and C ; namely, $k = \frac{315}{128}$ and $C = 0$.
-



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to NextStepMaths.com