

STEP II, 2009, Q3

3 Prove that

$$\tan\left(\frac{1}{4}\pi - \frac{1}{2}x\right) \equiv \sec x - \tan x. \quad (*)$$

(i) Use (*) to find the value of $\tan \frac{1}{8}\pi$. Hence show that

$$\tan \frac{11}{24}\pi = \frac{\sqrt{3} + \sqrt{2} - 1}{\sqrt{3} - \sqrt{6} + 1}.$$

(ii) Show that

$$\frac{\sqrt{3} + \sqrt{2} - 1}{\sqrt{3} - \sqrt{6} + 1} = 2 + \sqrt{2} + \sqrt{3} + \sqrt{6}.$$

(iii) Use (*) to show that

$$\tan \frac{1}{48}\pi = \sqrt{16 + 10\sqrt{2} + 8\sqrt{3} + 6\sqrt{6}} - 2 - \sqrt{2} - \sqrt{3} - \sqrt{6}.$$



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