STEP II, 2009, Q1 EC

Q1 The first question is usually intended to be a gentle introduction to the paper, and to allow all candidates to gain some marks without making great demands on either memory or technical skills. This year, however, and for the first time that I can recall in recent years, the obviously algebraic nature of the question was enough to deter half the candidates from attempting it. Indeed, apart from Q8, it was the least popular pure maths question. This was a great pity: the helpful structure really did guide folks in the right direction, and any half-decent candidates who did try it usually scored very highly on it. There were, nonetheless, a couple of stumbling-blocks along the way for the less wary, and many candidates tripped over them. The point at which most of the less successful students started to go astray was when asked to show that $ABCD$ is a rectangle. Lots of these folk elected to do so by working out distances … when the use of gradients would have been much simpler. The really disappointing thing was that many simply showed both pairs of opposite sides to be equal in length without realising that this only proved the quadrilateral a parallelogram. The next major difficulty was to be found in the algebra, in turning the area, $2(\alpha^2 - \beta^2)$, into something to do with u and v. It was quite apparent that many were unable to do so because they failed to appreciate that α and β were particular values of x and y that satisfy the two original curve equations, so that $\alpha^4 + \beta^4 = u$ and $\alpha\beta = v$. Then, squaring the area expression does the trick. Some got part of the way to grasping this idea, but approached from the direction of solving $x^4 + y^4 = u$ and $xy = v$ as simultaneous equations; the resulting surds-within-surds expressions for α and β were too indigestible for almost anyone to cope with.