

STEP II, 2009, Q13 MS

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- 13 For A: $p(\text{launch fails}) = p(>1 \text{ fail}) = 1 - p_0 - p_1 = 1 - q^4 - 4q^3p$
so that $E(\text{repair}) = \sum x p(x) = 0 \cdot q^4 + K \cdot 4q^3p + 4K(1 - q^4 - 4q^3p)$

$$= 4K[q^3p + (1 - q)(1 + q + q^2 + q^3) - 4q^3p]$$

$$= 4Kp[1 + q + q^2 - 2q^3]$$
- For B: $p(\text{launch fails}) = p(>2 \text{ fail}) = 1 - p_0 - p_1 - p_2 = 1 - q^6 - 6q^5p - 15q^4p^2$
so that $E(\text{repair}) = \sum x p(x)$

$$= 0 \cdot q^6 + K \cdot 6q^5p + 2K \cdot 15q^4p^2 + 6K(1 - q^6 - 6q^5p - 15q^4p^2)$$

$$= 6K[q^5p + 5q^4p^2 + (1 - q)(1 + q + q^2 + q^3 + q^4 + q^5) - 6q^5p - 15q^4p^2]$$
- Extracting the p and obtaining the remaining in terms of q only,

$$= 6Kp[q^5 + 5q^4(1 - q) + 1 + q + q^2 + q^3 + q^4 + q^5 - 6q^5 - 15q^4(1 - q)]$$

$$= 6Kp[1 + q + q^2 + q^3 - 9q^4 + 6q^5]$$
- Setting $\text{Rep}(A) = \frac{2}{3} \text{Rep}(B) \Rightarrow 12Kp[1 + q + q^2 - 2q^3] = 2Kp[1 + q + q^2 + q^3 - 9q^4 + 6q^5]$
Clearly, $p = 0$ is one solution and the rest simplifies to

$$0 = 3q^3(1 - 3q + 2q^2) = 3q^3(1 - q)(1 - 2q).$$
- We thus have $p = 1, 0, \frac{1}{2}$, with the 0 and 1 being rather trivial solutions.
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