STEP II, 2009, Q13 MS

13 For A: p(launch fails) = p(>1 fail) = $1 - p_0 - p_1 = 1 - q^4 - 4q^3p$

so that E(repair) = $\Sigma x\,\text{p}(x) = 0.q^4 + K.4q^3p + 4K(1 - q^4 - 4q^3p)$

$$= 4K\left[q^3p + (1-q)(1+q+q^2+q^3) - 4q^3p\right]$$

$$= 4Kp\left[1+q+q^2-2q^3\right]$$

For B: p(launch fails) = p(>2 fail) = $1 - p_0 - p_1 - p_2 = 1 - q^6 - 6q^5p - 15q^4p^2$

so that E(repair) = $\Sigma x\,\text{p}(x)$

$$= 0.q^6 + K.6q^5p + 2K.15q^4p^2 + 6K(1 - q^6 - 6q^5p - 15q^4p^2)$$

$$= 6K\left[q^5p + 5q^4p^2 + (1-q)(1+q+q^2+q^3+q^4+q^5) - 6q^5p - 15q^4p^2\right]$$

Extracting the p and obtaining the remaining in terms of q only,

$$= 6Kp\left[q^5 + 5q^4(1-q) + 1 + q + q^2 + q^3 + q^4 + q^5 - 6q^5 - 15q^4(1-q)\right]$$

$$= 6Kp\left[1+q+q^2+q^3-9q^4+6q^5\right]$$

Setting $\text{Rep}(A) = \frac{4}{3}\text{Rep}(B) \Rightarrow 12Kp\left[1+q+q^2-2q^3\right] = 2Kp\left[1+q+q^2+q^3-9q^4+6q^5\right]$

Clearly, $p = 0$ is one solution and the rest simplifies to

$$0 = 3q^3(1 - 3q + 2q^2) = 3q^3(1-q)(1-2q).$$

We thus have $p = 1, 0, \frac{1}{2}$, with the 0 and 1 being rather trivial solutions.