STEP II, 2009, Q11

11 A train consists of an engine and n trucks. It is travelling along a straight horizontal section of track. The mass of the engine and of each truck is M. The resistance to motion of the engine and of each truck is R, which is constant. The maximum power at which the engine can work is P.

Obtain an expression for the acceleration of the train when its speed is v and the engine is working at maximum power.

The train starts from rest with the engine working at maximum power. Obtain an expression for the time T taken to reach a given speed V, and show that this speed is only achievable if

$$P > (n+1)RV\,.$$

(i) In the case when $(n+1)RV/P$ is small, use the approximation $\ln(1-x) \approx -x - \frac{1}{2}x^2$ (valid for small x) to obtain the approximation

$$PT \approx \tfrac{1}{2}(n+1)MV^2$$

and interpret this result.

(ii) In the general case, the distance moved from rest in time T is X. *Write down*, with explanation, an equation relating P, T, X, M, V, R and n and hence show that

$$X = \frac{2PT - (n+1)MV^2}{2(n+1)R}\,.$$