STEP II, 2009, Q10 MS

10 The standard approach in collision questions is to write down the equations gained when applying the principles of *Conservation of Linear Momentum* (CLM) and *Newton's Experimental Law of Restitution* (NEL or NLR), and then what can be deduced from these.

For $P_{1,2}$: CLM $\Rightarrow m_1 u = m_1 v_1 + m_2 v_2$ and NEL $\Rightarrow eu = v_2 - v_1$.
Solving to determine the final speeds of P_1 and P_2 then yields

$$v_1 = \frac{(m_1 - em_2)}{m_1 + m_2}u \text{ and } v_2 = \frac{m_1(1+e)}{m_1 + m_2}u.$$

Similarly, for $P_{4,3}$: CLM $\Rightarrow m_4 u = m_4 v_4 + m_3 v_3$ and NEL $\Rightarrow eu = v_3 - v_4$, leading to

$$v_3 = \frac{m_4(1+e)}{m_3 + m_4}u \text{ and } v_4 = \frac{(m_4 - em_3)}{m_3 + m_4}u.$$

If we now write $X = OP_2$ and $Y = OP_3$ initially, and equate the times to the following collisions at O, we have

(1st collision): $\dfrac{(m_1 + m_2)X}{m_1(1+e)u} = \dfrac{(m_3 + m_4)Y}{m_4(1+e)u}$

and

(2nd collision): $\dfrac{(m_1 + m_2)X}{(m_1 - em_2)u} = \dfrac{(m_3 + m_4)Y}{(m_4 - em_3)u}$.

Cancelling u's and $(1 + e)$'s

$$\Rightarrow \frac{(m_1 + m_2)X}{m_1} = \frac{(m_3 + m_4)Y}{m_4} \text{ and } \frac{(m_1 + m_2)X}{(m_1 - em_2)} = \frac{(m_3 + m_4)Y}{(m_4 - em_3)}. \quad (*)$$

Dividing these two (or equating for X / Y) $\Rightarrow \dfrac{m_1 - em_2}{m_1} = \dfrac{m_4 - em_3}{m_4}$, which simplifies to

$\dfrac{m_2}{m_1} = \dfrac{m_3}{m_4}$. Finally substituting back into one of the equations **(*)** then gives

$$X\left(1 + \frac{m_2}{m_1}\right) = Y\left(1 + \frac{m_3}{m_4}\right) \Rightarrow X = Y.$$

Rather surprisingly, however, the momentum equations turn out to be totally unnecessary here. Consider …

Collision $P_{1,2}$: NEL $\Rightarrow eu = v_2 - v_1$

Collision $P_{4,3}$: NEL $\Rightarrow eu = v_3 - v_4$ so that $v_2 - v_1 = v_3 - v_4$ (*).

Next, the two equated sets of times are $\frac{X}{v_2} = \frac{Y}{v_3}$ and $\frac{X}{v_1} = \frac{Y}{v_4} \Rightarrow X v_3 = Y v_2$ and $X v_4 = Y v_1$.

Subtracting: $X(v_3 - v_4) = Y(v_2 - v_1) \Rightarrow X = Y$ from (*).
