

## STEP II, 2008, Q4

- 4 A curve is given by

$$x^2 + y^2 + 2axy = 1,$$

where  $a$  is a constant satisfying  $0 < a < 1$ . Show that the gradient of the curve at the point  $P$  with coordinates  $(x, y)$  is

$$-\frac{x + ay}{ax + y},$$

provided  $ax + y \neq 0$ . Show that  $\theta$ , the acute angle between  $OP$  and the normal to the curve at  $P$ , satisfies

$$\tan \theta = a|y^2 - x^2|.$$

Show further that, if  $\frac{d\theta}{dx} = 0$  at  $P$ , then:

- (i)  $a(x^2 + y^2) + 2xy = 0$ ;
- (ii)  $(1 + a)(x^2 + y^2 + 2xy) = 1$ ;
- (iii)  $\tan \theta = \frac{a}{\sqrt{1 - a^2}}$ .



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