STEP II, 2008, Q3

3 **(i)** Find the coordinates of the turning points of the curve $y = 27x^3 - 27x^2 + 4$. Sketch the curve and deduce that $x^2(1-x) \leqslant 4/27$ for all $x \geqslant 0$.

Given that each of the numbers a, b and c lies between 0 and 1, prove by contradiction that at least one of the numbers $bc(1-a)$, $ca(1-b)$ and $ab(1-c)$ is less than or equal to $4/27$.

(ii) Given that each of the numbers p and q lies between 0 and 1, prove that at least one of the numbers $p(1-q)$ and $q(1-p)$ is less than or equal to $1/4$.