STEP II, 2008, Q2

2 Let a_n be the coefficient of x^n in the series expansion, in ascending powers of x, of

$$\frac{1+x}{(1-x)^2(1+x^2)},$$

where $|x| < 1$. Show, using partial fractions, that either $a_n = n+1$ or $a_n = n+2$ according to the value of n.

Hence find a decimal approximation, to nine significant figures, for the fraction $\frac{11\,000}{8181}$.
[You are not required to justify the accuracy of your approximation.]