STEP II, 2008, Q11 MS

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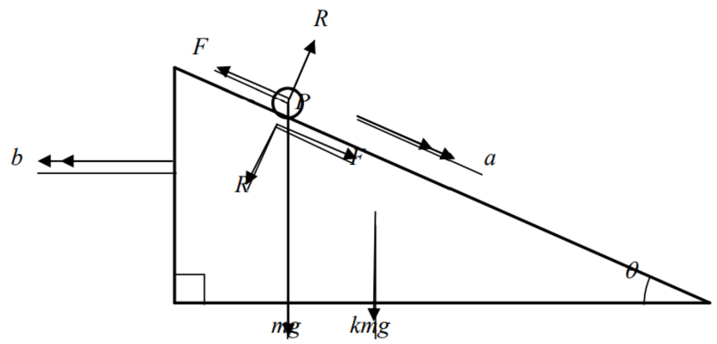

Once again, a good, clear diagram is an important starting-point, and the above diagram shows the relevant forces – labelled using standard notations – along with the accelerations of P down the sloping surface of the wedge (a) and the wedge itself along the plane (b).

(i) Noting the acceleration components of P are $a\cos\theta - b$ ($\rightarrow$) and $a\sin\theta$ ($\downarrow$), we employ *Newton's Second Law* as follows:

<u>*N2L* $\rightarrow$ *for P*</u> $\quad m(a\cos\theta - b) = R\sin\theta - F\cos\theta$

<u>*N2L* $\downarrow$ *for P*</u> $\quad ma\sin\theta = mg - F\sin\theta - R\cos\theta$

<u>*N2L* $\leftarrow$ *for wedge*</u> $\quad kmb = R\sin\theta - F\cos\theta$

From which it follows that $a\cos\theta - b = kb \Rightarrow b = \dfrac{a\cos\theta}{k+1}$.

Alternatively, one could use *N2L* to note *P*'s $\rightarrow$ accln. component and also the wedge's accln. $\leftarrow$, but instead use

<u>*CLM* $\leftrightarrow$</u> $\quad km\,bt = m\,(a\cos\theta - b)t$ (where t = time from release)

and this again leads to the above result for b.

Now, for P to move at 45° to the horizontal, $a\cos\theta - b = a\sin\theta$. Then

$$b = a(\cos\theta - \sin\theta) = \frac{a\cos\theta}{k+1}$$

$$\Rightarrow (k+1)(\cos\theta - \sin\theta) = \cos\theta \Rightarrow k + 1 - (k+1)\tan\theta = 1 \quad \text{and} \quad \tan\theta = \frac{k}{k+1}.$$

When $k = 3$, $\tan\theta = \frac{3}{4}$, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ and $b = \frac{1}{5}a$.

Substituting these into the first two equations of motion from (i), along with the use of the *Friction Law (in motion)*, which assumes that $F = \mu R$, gives

$$m(\tfrac{4}{5}a - b) = \tfrac{3}{5}R - \tfrac{4}{5}F \quad \text{or} \quad 3R - 4F = m(4a - 5b) = 3ma \Rightarrow R(3 - 4\mu) = 3ma$$

and

$$\tfrac{3}{5}ma = mg - \tfrac{3}{5}F - \tfrac{4}{5}R \quad \text{or} \quad 4R + 3F = m(5g - 3a) \Rightarrow R(4 + 3\mu) = 5mg - 3ma.$$

Dividing, or equating for R:

$$\frac{4+3\mu}{3-4\mu} = \frac{5g-3a}{3a} \Rightarrow (12 + 9\mu)a = 5(3 - 4\mu)g - (9 - 12\mu)a \Rightarrow a = \frac{5(3-4\mu)g}{3(7-\mu)}.$$

(ii) Finally, if $\tan\theta \le \mu$, then both P and the wedge remain stationary. So, technically, the answer is "nothing".