

STEP II, 2008, Q10

- 10 The lengths of the sides of a rectangular billiards table $ABCD$ are given by $AB = DC = a$ and $AD = BC = 2b$. There are small pockets at the midpoints M and N of the sides AD and BC , respectively. The sides of the table may be taken as smooth vertical walls.

A small ball is projected along the table from the corner A . It strikes the side BC at X , then the side DC at Y and then goes directly into the pocket at M . The angles BAX , CXY and DYM are α , β and γ respectively. On each stage of its path, the ball moves with constant speed in a straight line, the speeds being u , v and w respectively. The coefficient of restitution between the ball and the sides is e , where $e > 0$.

- (i) Show that $\tan \alpha \tan \beta = e$ and find γ in terms of α .
- (ii) Show that $\tan \alpha = \frac{(1+2e)b}{(1+e)a}$ and deduce that the shot is possible whatever the value of e .
- (iii) Find an expression in terms of e for the fraction of the kinetic energy of the ball that is lost during the motion.



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