STEP II, 2008, Q1

1 A sequence of points (x_1, y_1), (x_2, y_2), ... in the cartesian plane is generated by first choosing (x_1, y_1) then applying the rule, for $n = 1, 2, \ldots$,

$$(x_{n+1}, y_{n+1}) = (x_n^2 - y_n^2 + a,\ 2x_n y_n + b + 2)\,,$$

where a and b are given real constants.

(i) In the case $a = 1$ and $b = -1$, find the values of (x_1, y_1) for which the sequence is constant.

(ii) Given that $(x_1, y_1) = (-1, 1)$, find the values of a and b for which the sequence has period 2.