STEP II, 2007, Q6 MS

Q6 Once again, this starts off with a bit of very basic work that a realistic STEP candidate needs to be in a position to rattle off quickly and efficiently. The "Hence" at the start of line 2 of the question tells you that the answer to this integral is to be found in the two previous answers, without further calculus work being done. It is, therefore, very bad examination practice to ignore the "Hence" demand and go off on an "or otherwise" route that isn't actually needed. And there's a strong chance you may not get any marks at all for your alternative approach.

In (ii), there is no reason why you can't treat the given differential equation as a quadratic in $\frac{dy}{dx}$ and solve it to get two slightly different, and much simpler, differential equations than the original one. At this stage, if you have your wits about you, and you are **NOT** getting a $\sqrt{3+x^2}$ anywhere in sight, then you really ought to be a bit suspicious about why not! For the rest of it, it is a simple case of integrating using (i)'s result, and then applying the given initial condition to find the constants of integration in each of the two cases.

Answers: (i) $\frac{1}{\sqrt{3+x^2}}$, $\frac{2x^2+3}{\sqrt{3+x^2}}$; $\frac{1}{2}x\sqrt{3+x^2} + \frac{3}{2}\ln\left(x+\sqrt{3+x^2}\right)$ $(+C)$.

(ii) $y = \frac{1}{6}x\sqrt{3+x^2} + \frac{1}{2}\ln\left(x+\sqrt{3+x^2}\right) - \frac{1}{6}x^2 - \frac{1}{6} - \frac{1}{2}\ln 3$;

$y = -\frac{1}{6}x\sqrt{3+x^2} - \frac{1}{2}\ln\left(x+\sqrt{3+x^2}\right) - \frac{1}{6}x^2 + \frac{1}{2} + \frac{1}{2}\ln 3$.