

STEP II, 2007, Q5

5 In this question, $f^2(x)$ denotes $f(f(x))$, $f^3(x)$ denotes $f(f(f(x)))$, and so on.

(i) The function f is defined, for $x \neq \pm 1/\sqrt{3}$, by

$$f(x) = \frac{x + \sqrt{3}}{1 - \sqrt{3}x}.$$

Find by direct calculation $f^2(x)$ and $f^3(x)$, and determine $f^{2007}(x)$.

(ii) Show that $f^n(x) = \tan(\theta + \frac{1}{3}n\pi)$, where $x = \tan \theta$ and n is any positive integer.

(iii) The function $g(t)$ is defined, for $|t| \leq 1$ by $g(t) = \frac{\sqrt{3}}{2}t + \frac{1}{2}\sqrt{1-t^2}$. Find an expression for $g^n(t)$ for any positive integer n .



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