STEP II, 2007, Q4 MS

Q4 This was actually not a particularly popular, or well done, question, although I still maintain that it is quite an easy one when it comes down to it! To begin with, it is really, *really* obvious that you need to expand the given trig. expressions using the *Addition Formulae*. Then, in order to obtain tans throughout, rather than sines and cosines, you are going to have to divide by ….. (hint: note the introductory conditions at the very start of the question, which are given to enable you not to worry about dividing by …..). Wangling it into the given form and checking that the given condition holds is not much more than an algebraic exercise at this stage, and shouldn't prove too much of a burden. However, it is easy to overlook the fact that you are asked to prove an "if and only if" statement, which is two-directional. In point of fact, it is the case here that a clear line of reasoning from first equation to final one actually *is* entirely reversible, although it is best to (at least) point out that this is so, rather than ignore it.

For the next three parts, see how this result can now be used to solve each of the given equations, once the "A" and the "B" have been clearly identified. Also, don't forget to identify the α, β and γ (the same in each of the three parts) and verify that $\alpha^2 = \beta^2 + \gamma^2$. It is, of course, perfectly possible to start each bit from scratch, and the wording of the question doesn't actually prevent you from doing so, but it would seem a bit of a waste of time and effort to do so. Having said that, several candidates successfully did (iii) by collecting the two ($3x$) terms up together and collecting them up in an $R\sin(3x + \theta)$ form.

Incidentally, my favourite part of the question was (ii), in which I got to play a bit of a dirty trick – the statement looks like an equation, but is actually an ….. because …..

Answers: (i) $\frac{2\pi}{3}, \frac{5\pi}{3}$; (ii) all $x \in [0, 2\pi)$; (iii) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ and $\frac{\pi}{3}, \frac{4\pi}{3}$.