STEP II, 2007, Q4

4 Given that $\cos A$, $\cos B$ and β are non-zero, show that the equation

$$\alpha\sin(A-B)+\beta\cos(A+B)=\gamma\sin(A+B)$$

reduces to the form

$$(\tan A-m)(\tan B-n)=0\,,$$

where m and n are independent of A and B, if and only if $\alpha^2=\beta^2+\gamma^2$.

Determine all values of x, in the range $0\leqslant x<2\pi$, for which:

(i) $2\sin(x-\frac{1}{4}\pi)+\sqrt{3}\cos(x+\frac{1}{4}\pi)=\sin(x+\frac{1}{4}\pi)$;

(ii) $2\sin(x-\frac{1}{6}\pi)+\sqrt{3}\cos(x+\frac{1}{6}\pi)=\sin(x+\frac{1}{6}\pi)$;

(iii) $2\sin(x+\frac{1}{3}\pi)+\sqrt{3}\cos(3x)=\sin(3x)$.