STEP II, 2007, Q3 MS

Q3 The first part of this question is a standard piece of bookwork, and requires only a modest ability to cope with substitution integration and a bit of trig. identity work. In (i) (a), you need to spot a suitable substitution for yourself – comparing the integrand with that in the introductory bit gives the game away, if you're stuck. In my day, the $t = \tan \frac{1}{2}x$ substitution was a very common bit of work, but you don't see it very often at A-level nowadays, so you could be forgiven for not being entirely familiar with it. Nonetheless, the principles of substitution still apply, and there may be the odd trig. identity to be employed, of course. The final two pieces of work here are greatly eased by the fact that they can be done in either direction. By that, I mean that one can eliminate all the ts in favour of xs, or vice versa. If you successfully complete part (i) (b), then (ii) is so much easier, since the only difference is that you must have $3\sin^2 x$ in the denominator to give $14t^2$ instead of $6t^2$. Another simple substitution then changes the form into a standard arctan integral and, with a little bit of care, the whole thing can be wrapped up quite smoothly.

Overall, I would suggest that this is a fairly routine question, with no great leaps of thought required for a good A-level candidate to be able to work their way through it. What *is* required, however, is a high level of thoroughness and familiarity with the basic techniques of the trig. and calculus involved therein. Such capabilities are an essential requirement if you are preparing for future STEPs.

Answers: (i) (a) $\frac{\pi}{4}$; (ii) $\frac{\pi}{6\sqrt{3}}$.