

STEP II, 2007, Q3

3 By writing $x = a \tan \theta$, show that, for $a \neq 0$, $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + \text{constant}$.

(i) Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} dx$.

(a) Evaluate I .

(b) Use the substitution $t = \tan \frac{1}{2}x$ to show that $\int_0^1 \frac{1 - t^2}{1 + 6t^2 + t^4} dt = \frac{1}{2}I$.

(ii) Evaluate $\int_0^1 \frac{1 - t^2}{1 + 14t^2 + t^4} dt$.



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