STEP II, 2007, Q3

3 By writing $x = a\tan\theta$, show that, for $a \neq 0$, $\displaystyle\int \frac{1}{a^2+x^2}\,\mathrm{d}x = \frac{1}{a}\arctan\frac{x}{a} + \text{constant}$.

(i) Let $\displaystyle I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}\,\mathrm{d}x$.

(a) Evaluate I.

(b) Use the substitution $t = \tan\frac{1}{2}x$ to show that $\displaystyle\int_0^1 \frac{1-t^2}{1+6t^2+t^4}\,\mathrm{d}t = \frac{1}{2}I$.

(ii) Evaluate $\displaystyle\int_0^1 \frac{1-t^2}{1+14t^2+t^4}\,\mathrm{d}t$.