STEP II, 2007, Q1 MS

Q1 It is important to get off to a good start in any examination, especially so in STEPs, and Q1 is specifically designed to get as many candidates as possible off to such a start. Binomial series expansions are given in any of the permitted formulae books, and there is really no excuse for failing to pick up the marks on the introductory bit of the question. It is almost certainly to your advantage to simplify the terms of the expansion, but a little bit of care is in order here, else you are automatically losing accuracy marks later on in (a) and (b).

For part (a), you are told exactly what value of k to choose, and it is simply a case of using it **on both sides of the statement** – in the LHS to show that you can extract a sensible multiple of $\sqrt{3}$, and then in the RHS to see what you get as a decimal. Remember that working in powers of 10 makes the numerical working a lot simpler.

In (b), you have to choose a suitable value of k so that the LHS gives a multiple of $\sqrt{6}$. There is a small (but negative) integer value of k which will do this nicely. Many candidates, however, actually chose to work with $k = 50$ and, if you check, you will see that this *seems* to work equally well. However, the approximation gained is not nearly so accurate; this is because ….. ? Also, not a few candidates chose values of k greater than 100 in absolute value, and these are even worse, because ….. ?

In (ii), it is certainly possible to work back from the final answer in order to figure out what value of k to use here, but (again) you are looking for some (presumably) integer value that will this time yield a perfect cube multiple of 3 when $1+\dfrac{k}{1000}$ is written as an improper fraction.

For interest's sake, the original version of the question used the first *three* terms of the series expansion with $k = 24$ to find an approximation to $\sqrt[3]{2}$.

Answers: (i) $1+\dfrac{k}{200}-\dfrac{k^2}{80\,000}+\dfrac{k^3}{16\,000\,000}$; (a) 1.732 05; (b) 2.449 49.