

STEP II, 2007, Q13

- 13 Given that $0 < r < n$ and r is much smaller than n , show that $\frac{n-r}{n} \approx e^{-r/n}$.

There are k guests at a party. Assuming that there are exactly 365 days in the year, and that the birthday of any guest is equally likely to fall on any of these days, show that the probability that there are at least two guests with the same birthday is approximately $1 - e^{-k(k-1)/730}$.

Using the approximation $\frac{253}{365} \approx \ln 2$, find the smallest value of k such that the probability that at least two guests share the same birthday is at least $\frac{1}{2}$.

How many guests must there be at the party for the probability that at least one guest has the same birthday as the host to be at least $\frac{1}{2}$?



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