STEP II, 2007, Q12 MS

Q12 With given answers, as here, it is important to make your method clear, since there is a lot of fiddling going on as candidates inevitably manage to wangle this answer somehow. Splitting the required event into a series of mutually exclusive events and recognising which of these events are independent, is crucial, and it helps both you and the examiner if there is an accompanying (brief!) explanation as to what you are doing. It seems to me that the first part can be approached in at least a couple of obvious ways. Firstly, one could work out the prob. that one die gives at least one 6 in the first r throws, $P(r)$, say, and then observing that the prob. that both dice have given a 6 **at** the r^{th} throw is $P(r) - P(r-1)$. Alternatively, one could write it as the sum of the probs. that {neither dice has recorded a 6 in the first $r-1$ throws and then both give 6s} with the prob. that {one die gives a 6 before the r^{th} throw and then the 2^{nd} die first gives a 6 on the r^{th} throw}.

Finding the expected value of the number of throws is routine, in principle at least, and you are given a result to use to help you with this, if needed. In (ii), equating this expression (in terms of p only) to m and then re-arranging gives a ….. equation in p, which should now be very familiar territory.

Answers: (i) $p = \frac{1}{m}\left\{m+1-\sqrt{m^2-m+1}\right\}$.