STEP II, 2007, Q11

11 *In this question take the acceleration due to gravity to be* $10\,\mathrm{m\,s^{-2}}$ *and neglect air resistance.*

The point O lies in a horizontal field. The point B lies 50 m east of O. A particle is projected from B at speed $25\,\mathrm{m\,s^{-1}}$ at an angle $\arctan\frac{1}{2}$ above the horizontal and in a direction that makes an angle 60° with OB; it passes to the north of O.

(i) Taking unit vectors **i**, **j** and **k** in the directions east, north and vertically upwards, respectively, find the position vector of the particle relative to O at time t seconds after the particle was projected, and show that its distance from O is

$$5(t^2 - \sqrt{5}t + 10)\,\mathrm{m}.$$

When this distance is shortest, the particle is at point P. Find the position vector of P and its horizontal bearing from O.

(ii) Show that the particle reaches its maximum height at P.

(iii) When the particle is at P, a marksman fires a bullet from O directly at P. The initial speed of the bullet is $350\,\mathrm{m\,s^{-1}}$. Ignoring the effect of gravity on the bullet show that, when it passes through P, the distance between P and the particle is approximately 3 m.