STEP II, 2006, Q9 MS

Q9 The most important thing you can do on a question like this, is to draw a good, decent-sized diagram first, marking on it all the relevant forces. In fact, since a lot of extra forces come into play in the second part of the question, a completely new diagram here is pretty much essential. It is also helpful to have the painter, P, in a general position on the ladder; say, a distance xa from its base up along it. [Note that xa is so much better than x, so that – since all distances are now multiples of a – these will cancel in the moments equation and make things *look* simpler.] Now resolve twice and take moments (easier about the base of the ladder), and use the *Friction Law* (in its inequality form, since we don't need to know when it attains its maximum). And then sort out the remaining algebra. On this occasion, it is not unreasonable to assume that P is at the top of the ladder when slipping is most likely, and go from there.

In (ii), the extra forces involved are the weight of the table, the reaction forces between its legs and the ground **and** the reaction of the ladder's base on the table (previously ignored when the ladder was on the ground). The standard approach now is to assume that the system is rotationally stable and see when slipping occurs; then to assume that the system is translationally stable and see when tilting occurs. Again, this involves resolving twice and taking moments; using the *Friction Law* – with equilibrium broken when one of the reactions between table and ground is zero – and deciding which, if any, happens first.

Answers: Table slips on ground when P is distance $5a$ up the ladder. Table turns about edge furthest from the wall when P is distance $\frac{11}{3}a$ up the ladder. Thus, tilting occurs first.