STEP II, 2006, Q8 MS

Q8 I'm afraid that this question involves but a single idea: namely, that of intersecting lines. The first two parts are simple "bookwork" tasks, requiring nothing more than an explanation of the vector form of a line equation as $\mathbf{r}$ = p.v. of any point on the line + some scalar multiple of any vector (such as $\mathbf{y} - \mathbf{x}$, in this case) parallel to the line; then the basic observation that $CB \parallel OA \Rightarrow \overrightarrow{CB} = \lambda\mathbf{a}$ to justify the second result.

Thereafter, it is simply a case, with (admittedly) increasingly complicated looking position vectors coming into play, of equating **a**'s and **c**'s in pairs of lines to find out the position vector of the point where they intersect. If the final part is to be answered numerically, then the parameter λ must cancel somewhere before the end.

Answers: (ii) $\mathbf{d} = \left(\frac{1}{1-\lambda}\right)\mathbf{c}$; $\mathbf{e} = \frac{1}{3}\mathbf{a}$; $\mathbf{f} = \mathbf{c} + \frac{1}{2}\lambda\mathbf{a}$; $\mathbf{g} = \left(\frac{2\lambda}{2+3\lambda}\right)\mathbf{a} + \left(\frac{2}{2+3\lambda}\right)\mathbf{c}$; $\mathbf{h} = \frac{2}{5}\mathbf{a}$.

Thus $OH : HA = 2 : 3$ (as H lies two-fifths of the way along the line OA).