

## STEP II, 2006, Q8

- 8 Show that the line through the points with position vectors  $\mathbf{x}$  and  $\mathbf{y}$  has equation

$$\mathbf{r} = (1 - \alpha)\mathbf{x} + \alpha\mathbf{y},$$

where  $\alpha$  is a scalar parameter.

The sides  $OA$  and  $CB$  of a trapezium  $OABC$  are parallel, and  $OA > CB$ . The point  $E$  on  $OA$  is such that  $OE : EA = 1 : 2$ , and  $F$  is the midpoint of  $CB$ . The point  $D$  is the intersection of  $OC$  produced and  $AB$  produced; the point  $G$  is the intersection of  $OB$  and  $EF$ ; and the point  $H$  is the intersection of  $DG$  produced and  $OA$ . Let  $\mathbf{a}$  and  $\mathbf{c}$  be the position vectors of the points  $A$  and  $C$ , respectively, with respect to the origin  $O$ .

- (i) Show that  $B$  has position vector  $\lambda\mathbf{a} + \mathbf{c}$  for some scalar parameter  $\lambda$ .
- (ii) Find, in terms of  $\mathbf{a}$ ,  $\mathbf{c}$  and  $\lambda$  only, the position vectors of  $D$ ,  $E$ ,  $F$ ,  $G$  and  $H$ . Determine the ratio  $OH : HA$ .



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