STEP II, 2006, Q7 MS

Q7 This is a reasonably routine question to begin with. The general gradient to the curve can be found by differentiating either implicitly or parametrically. Finding the gradient and equation of line *AP* is also standard enough; as is setting $y = b$ in order to find the coordinates of *Q*: $\left(\frac{(1-k)a}{(1+k)}, b\right)$. The equation of line *PQ* follows a similar line of working, to get $y = \left(\frac{-(1-k^2)b}{2ka}\right)x + \frac{b(1+k^2)}{2k}$. If you are not familiar with the $t = \tan\frac{1}{2}$-angle identities, the next part should still not prove too taxing, as you should be able to quote, or derive (from the formula for $\tan(A + B)$ in the formula books), the formula for $\tan 2A$ soon enough; and the widely known, "*Pythagorean*", identity $\text{cosec}^2 A = 1 + \cot^2 A$ will help you sort out the gradient and intercept of *PQ* to show that the two forms of this line are indeed the same when $k = \tan(\frac{1}{2}\alpha)$.

A sketch of the ellipse, though not explicitly asked-for, should be made (at least once) so that you can draw on the lines *PQ* in the cases $k = 0$ and $k = 1$.

Answers: Yes; *PQ* is the vertical tangent to the ellipse.
Yes; *PQ* is the horizontal tangent to the ellipse.

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