

## STEP II, 2006, Q7

- 7 An ellipse has equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . Show that the equation of the tangent at the point  $(a \cos \alpha, b \sin \alpha)$  is

$$y = -\frac{b \cot \alpha}{a} x + b \operatorname{cosec} \alpha.$$

The point  $A$  has coordinates  $(-a, -b)$ , where  $a$  and  $b$  are positive. The point  $E$  has coordinates  $(-a, 0)$  and the point  $P$  has coordinates  $(a, kb)$ , where  $0 < k < 1$ . The line through  $E$  parallel to  $AP$  meets the line  $y = b$  at the point  $Q$ . Show that the line  $PQ$  is tangent to the above ellipse at the point given by  $\tan(\alpha/2) = k$ .

Determine by means of sketches, or otherwise, whether this result holds also for  $k = 0$  and  $k = 1$ .



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