STEP II, 2006, Q6 MS

Q6 The two vectors to be used are clearly $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. The inequality arises when you note that $\cos^2\theta \leq 1$. The statement is an equality (equation) when $\cos\theta = \pm 1$, in which case the two vectors must be parallel, so that one is a (non-zero) multiple of the other. [The question cites an example of a result widely known as the *Cauchy-Schwarz Inequality*.] The equality case of the inequality is then used in the two following parts; simply in (i) – since we must have $y = z = \ldots\ldots$, from which it follows that $x = \frac{1}{2}$ this. In (ii), you should check that this is indeed an equality case of the inequality when the two vectors are ….. and ….. The parallel condition (one being a multiple of the other) now gives p, q and r in terms of some parameter (say λ), and you can substitute them into the linear equation (of the two given this is clearly the more straightforward one to use), find λ, and then deduce p, q and r; these values actually being unique.

Answers: $x = \lambda a$, $y = \lambda b$ and $z = \lambda c$; (i) $x = 7$; (ii) $p = 24$, $q = 6$, $r = 1$.