

STEP II, 2006, Q6

- 6 By considering a suitable scalar product, prove that

$$(ax + by + cz)^2 \leq (a^2 + b^2 + c^2)(x^2 + y^2 + z^2)$$

for any real numbers a, b, c, x, y and z . Deduce a necessary and sufficient condition on a, b, c, x, y and z for the following equation to hold:

$$(ax + by + cz)^2 = (a^2 + b^2 + c^2)(x^2 + y^2 + z^2).$$

- (i) Show that $(x + 2y + 2z)^2 \leq 9(x^2 + y^2 + z^2)$ for all real numbers x, y and z .
- (ii) Find real numbers p, q and r that satisfy both

$$p^2 + 4q^2 + 9r^2 = 729 \quad \text{and} \quad 8p + 8q + 3r = 243.$$



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