# STEP II, 2006, Q4 MS

**Q4** Using the given substitution, the initial result is established by splitting the integral into its two parts, and then making the simple observation that $\int_0^\pi x\,\mathrm{f}(\sin x)\ \mathrm{d}x = \int_0^\pi t\,\mathrm{f}(\sin t)\,\mathrm{d}t$ .

This result is now used directly in (i), along with a substitution (such as $c = \cos x$). The resulting integration can be avoided by referring to your formula book, or done by using partial fractions. In (ii), the integral can be split into two; one from 0 to π, the second from π to 2π. The first of these is just (i)'s integral, and the second can be determined by using a substitution such as $y = x - \pi$ (the key here is that the limits will then match those of the initial result, which you should be looking to make use of as much as possible). In part (iii), the use of the double-angle formula for $\sin 2x$ gives an integral involving sines and cosines, but this must also count as a function of $\sin x$, since $\cos x = \sqrt{1 - \sin^2 x}$ . Thus the initial result may be applied here also. Once again, the substitution $c = \cos x$ reduces the integration to a standard one.

**Answers:** (i) $\frac{1}{4}\pi \ln 3$ ; (ii) $-\frac{1}{2}\pi \ln 3$ ; (iii) $\pi \ln \frac{4}{3}$ .