STEP II, 2006, Q4

4 By making the substitution $x = \pi - t$, show that

$$\int_0^{\pi} x\mathrm{f}(\sin x)\mathrm{d}x = \tfrac{1}{2}\pi \int_0^{\pi} \mathrm{f}(\sin x)\mathrm{d}x \,,$$

where $\mathrm{f}(\sin x)$ is a given function of $\sin x$.

Evaluate the following integrals:

(i) $\displaystyle\int_0^{\pi} \frac{x\sin x}{3+\sin^2 x}\,\mathrm{d}x\,;$

(ii) $\displaystyle\int_0^{2\pi} \frac{x\sin x}{3+\sin^2 x}\,\mathrm{d}x\,;$

(iii) $\displaystyle\int_0^{\pi} \frac{x|\sin 2x|}{3+\sin^2 x}\,\mathrm{d}x\,.$