STEP II, 2006, Q1 EC

1 Almost all candidates attempted this question and most managed at least some measure of success; although the high level of algebra required to see matters through to a successful conclusion proved to be a decisive factor in whether attempts got much over half-marks. A minority of candidates worked with u_n and u_{n+r} (for the appropriate r's) and thereby made the algebra rather harder for themselves; whereas it had been intended that they should work with u_1 (with the given value of 2) and the appropriate u_r in order to determine periodicity. The other major problem arose when candidates worked backwards from (say) u_5 towards u_1, rather than forwards. This often generated nested sets of bracketed expressions of the form

$$u_5 = k - \cfrac{36}{k - \cfrac{36}{k - \frac{36}{k - \ldots\ldots}}}$$

which only the hardiest were able to unravel successfully; while a forwards approach would have found each of $u_2, u_3, \ldots$ successively as much simpler (rational) terms.

Another common error arose when candidates failed to note that, if $k = 20$ gives a constant sequence, then, for a sequence of period 2, the answers "$k = 20$ and 0" can't both be correct. Similarly, for a sequence of period 4, the values 0 and 20 should appear as possible solutions when equating u_5 to u_1, but should be discounted. Whilst many candidates noted these points – and some shrewdly used their existence to help factorise the arising quartic equation in k – it is still clearly the case that a large proportion of A-level students, even the better ones, are happy to assume that any solution to an equation they end up having to solve is valid, irrespective of the context of the underlying problem or the logic of their work (viz. *necessary and/or sufficient conditions*).

Although only the most basic of arguments was required to establish that $u_n \geq 2$ at the beginning of part (ii), it was clear that most candidates were really not comfortable handling inequalities, and lacked practice in constructing reasonable mathematical arguments. Far too many failed to work generally at all, and simply showed that the first few terms were greater than or equal to 2, concluding with a waffle-y "etc., etc., etc." sort of argument. In the very last part, it was important to appreciate that a limit is approached when successive terms effectively become the same. No formal work beyond this simple idea was required, and the resulting quadratic gave two solutions, only one of which was greater than 2. Rather a lot of candidates were happy with this idea and rattled it through very quickly.