STEP II, 2006, Q14 MS

Q14 For $y = \dfrac{1}{x\ln x}$, $y \to -\infty$ as $x \to 0$ and $y \to 0$ (+ve) as $x \to \infty$ are the obvious asymptotic tendencies of the graph. Since $\ln 1 = 0$, there is also a discontinuity at $x =$ 1, and you must decide what happens to the graph either side of this point.

For the rest of the question, its is essential to be able to integrate $\dfrac{1}{x\ln x}$. This can be done either by the sneaky observation that it can be written in the form $\dfrac{1/x}{\ln x}$, so that the numerator is exactly the derivative of the denominator – a standard log. integral form – or by using a substitution such as $u = \ln x$.

In (i) and (ii), it is now just a case of substituting in the limits and sorting out the log. work. Having gained the answer for (ii), in log. form, the numerical approximation arises from using the first few terms of the series, given in the formula books, for $\ln(1 + x)$ with $x = \ldots$..

In the very final part, a range is given that turns out to be outside the non-zero part of the *pdf*. A little bit of work needs to be done to justify this, and then you can write down the answer.

Answers: (i) $\lambda = \dfrac{1}{\ln\frac{1}{2}}$ or $-\dfrac{1}{\ln 2}$; (iv) 0.