STEP II, 2006, Q13 MS

Q13 To be honest, this was more of a counting question than anything, at least to begin with. Although it is possible to attack (i) by multiplying and adding various probabilities, it is most easily approached by examining the 24 permutations of {1, 2, 3, 4} individually, and seeing what choice is made in each case. To make life easy for yourself, be systematic in listing these possibilities.

This example should point you in the right direction, but don't be tempted to just write down the answer that you've spotted without any justification for how it arises ***in the general case***. To begin with, deal with what happens when the largest cone is offered first; then the second-largest being first; then the third. By this stage it should be easy to justify the general case as to what happens when the r^{th} largest cone is the first to be offered – then the largest is chosen if it appears first of the remaining $(r-1)$ cones that are bigger than the r^{th}. With probability …..

Answers: (i) $P_4(2) = \frac{7}{24}$; $P_4(3) = \frac{4}{24}$ or $\frac{1}{6}$; $P_4(1) = \frac{2}{24}$ or $\frac{1}{12}$;

(ii) $\frac{1}{n}\left\{0+1+\frac{1}{2}+\frac{1}{3}+\ldots..+\frac{1}{n-1}\right\}$ or $\frac{1}{n}\sum_{r=1}^{n-1}\frac{1}{r}$