STEP II, 2006, Q11 MS

Q11 The equations of motion in the x- and y-directions can be found by integrating up from accelerations, or by using the *constant-acceleration formulae*. Setting $y = 0$ gives $t = 0$ or $t = \ldots..$ (as usual). Substituting this into the expression for x then gives the distance OA.

In (i), the time when $\dot{x} = 0$ must occur before the time found above. This gives an inequality involving sine and cosine, which can be simplified to give the tangent of the angle required.

In (ii), OB is just OA with $\theta = 45^\circ$. Then OA is maximised either by calculus (a little trickier here) or by using the double-angle formulae for sines and cosines and then working with an expression of the form $a \cos 2\theta + b \sin 2\theta + c$, for which there is a standard piece of work to yield the form $R \cos(2\theta - \emptyset) + c$, which has an obvious maximum of $R + c$ (with R here being in terms of f and g).

For the very last part, $f = g$ with $\theta = 45^\circ$ gives $x = y$ for B's motion, and the particle moves up, and then down, a straight line inclined at 45° to the horizontal, to land at its original point of projection.

Answers: (i) $\alpha = \arctan\left(\frac{g}{2f}\right)$; (ii) answer as above.