STEP II, 2005, Q9 MS

Q9(i) To begin with it is essential to draw a complete force diagram without omission or duplication of forces. In this respect, no particular direction, such as the horizontal or up the plane, should be assumed for the action of P. A convenient specification for the direction of P is $\theta + \pi/6$ with the horizontal, where at this stage, $0 < \theta < \pi/2$, but otherwise is general.

For motion up the slope it is necessary that

$$P\cos\theta \geq mg + mg/2 + (1/2\sqrt{3})(mg\sqrt{3}/2) + (1/\sqrt{3})(mg\sqrt{3} - P\sin\theta).$$

From this inequality it follows that $P\cos(\theta - \pi/6) \geq (11\sqrt{3}/8)mg$ so that P_{min} is defined by $\theta = \pi/6$ and hence is equal to $(11\sqrt{3}/8)mg$.

Thus P_{min} acts at a direction making an angle of $\pi/3$ above the horizontal.

(ii) In order to clarify ideas in the second part of this question, it is advisable to draw a separate diagram. The friction, which again is limiting, acts up the slope so that now

$$P\cos\theta \geq mg + mg/2 - (1/2\sqrt{3})(mg\sqrt{3}/2) - (1/\sqrt{3})(mg\sqrt{3} - P\sin\theta)$$

which implies $P\cos(\theta + \pi/6) \geq (\sqrt{3}/8)mg$. Thus in this case, $P_{min} = (\sqrt{3}/8)mg$ and is achieved when $\theta = -\pi/6$, so that P_{min} acts horizontally.