STEP II, 2005, Q9 EC

Q9 Few responses showed a clear and complete diagram for both parts of this question and no doubt this deficiency was the main cause of bad modelling. Candidates generally needed to think through the application of Newton's laws of motion in this context in order to ensure that all terms were present and this is what some failed to do.

(i) A particular direction of P, usually parallel to the slope, was assumed in about half of all responses. Such a presupposition trivialised the question and so very little credit could be given for this strategy.

In contrast, the best responses went very rapidly to the fundamental equation

$$P\cos\theta = mg + mg/2 + (1/2\sqrt{3})(mg\sqrt{3}/2) + (1/\sqrt{3})(mg\sqrt{3} - P\sin\theta).$$

where $\theta + \pi/6$ is the angle which P makes with the horizontal. Here, calculus methods were often used in order to determine the optimal direction of P, in the sense of the question, and very often such arguments were incomplete and almost incoherent.

The superior method of first writing

$$P\sin(\theta + \pi/3) = (11\sqrt{3}/8)mg$$

followed by the use of $\sin(.) \leq 1$ was not to be seen to any great extent and this would suggest that lack of facility in the working of trigonometric expressions was a partial cause of failure in this question.

(ii) Much the same comments carry over to the situation here where the critical equation is

$$P\cos\theta = mg + mg/2 - (1/2\sqrt{3})(mg\sqrt{3}/2) - (1/\sqrt{3})(mg\sqrt{3} - P\sin\theta).$$

However, few responses at this stage were complete and correct.

(i) Direction for least magnitude of P is parallel to the slope.

(ii) Least magnitude of P is $(\sqrt{3}/8)mg$.