STEP II, 2005, Q8 EC

Q8 This question led to application of essentially correct methodologies. However, at the technical level there many inaccuracies.

Most responses soon arrived at something like $A - 1/y = \int x^3(1+x^2)^{-5/2}\,dx$.

At the integration stage, at least six methods were in evidence. The most popular of these was based on the integration by parts rule. This obvious and simple strategy was generally applied accurately. Also in evidence were the use of substitutions such as $u = x^2$, $v = 1 + x^2$, $x = \tan t$, $x = \sinh w$ and a consideration of the derivative of $1/y = (2 + 3x^2)/[3(1 + x^2)^{3/2}]$.

This last method would have been completely acceptable had the solution not been displayed in the question. In this context, however, it must necessarily be regarded as verification and, as such, did not merit full credit.

Application of the given initial condition was usually accurate if only because the displayed result was at hand.

The large positive x approximation was established in a rigorous way by only a minority. Following the expansion of $(1 + x^2)^{-3/2}$, terms were prematurely discarded and then brought back into the working again in some illegal way so as to produce the displayed approximation.

The sketch of C was often deficient in some way in that the gradient at (0,1) was non-zero and the location of the horizontal asymptote was not identified.

A substantial minority of candidates failed to observe that the second differential equation can be obtained immediately from the first by the substitution $y = z^2$. Such a deduction makes the drawing of the second diagram, involving 2 branches, a simple matter.