STEP II, 2005, Q7 EC

Q7 This was the least well answered question of Section A and this was due mainly to a lack of understanding of the geometry supplemented by defective expertise in dealing with inequalities involving trigonometric functions.

(i) The description of the locus of P was usually correct as far as it went. Basically, what was required was the identification of it as a circle, the radius of this circle, the location of the centre and the plane in which it is contained. The same is required for the locus of Q, but here, some candidates thought that it is an ellipse and were usually unable to specify the plane in any geometrically intelligible way.

(ii) Most responses showed the correct formation of a relevant scalar product in terms of t. Thereafter there was some illegal working of the trigonometry based on the incorrect resolution of $\cos(.)\cos(..)$ into a sum of cosines and $\sin(.)\sin(..)$ into a difference of cosines. Moreover much energy was expended on finding OQ even when the candidate had already concluded in (i) that it is constant and equal to 3, and even more remarkably, the proving that $OQ = 3$ did not lead to any correction of the statement in (i) that Q describes an ellipse.

(iii) Generally, failure in (ii) did not inhibit sensible attempts at this concluding section of the question. Nevertheless, few candidates seemed to realise that 1 cycle in the $t-$ domain corresponds to 2 cycles in the $\theta-$ domain. Certainly candidates would have helped themselves considerably if they had worked from an accurate sketch graph of the cosine function. As it was, very few did this and for the most part fell back on hazy inequality arguments which, more often than not, were inconsistent with basic properties such as $\cos\theta$ is decreasing over the open interval $(0, \pi)$ and is increasing over the open interval $(\pi, 2\pi)$.

(i) P describes a circle centre O and radius 1 in the $x - y$ plane.

Q describes circle centre O and radius 3 in the plane $\sqrt{3}x - z = 0$.