

STEP II, 2005, Q6 MS

Q6 (i) The power series representations of $(1+x)^{-k}$ for $k = 1, 2, 3$ are standard and should be well known by any candidate for this examination. In fact the general terms of these three series are x^n , $(n+1)x^n$, $(1/2)(n+1)(n+2)x^n$, respectively.

The displayed series may be summed by using the general terms obtained. Thus,

$$\sum_{n=1}^{\infty} n2^{-n} = (1/2)(1 - 1/2)^{-2} = 2,$$

and as $\sum_{n=1}^{\infty} n(n+1)2^{-n} = 8$, then $\sum_{n=1}^{\infty} n^2 2^{-n} = 8 - 2 = 6$.

(ii) The obtaining of the general term of the power series (*) for $(1-x)^{-1/2}$ is a straightforward application of the binomial series for a general exponent.

To sum the penultimate series, put $x = 1/3$ in (*) and to sum the final series, first differentiate (*) with respect to x and then put $x = 1/3$. The sums will be found to be $\sqrt{3/2}$ and $(1/4)\sqrt{3/2}$, respectively.



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