

## STEP II, 2005, Q5

- 5 The angle  $A$  of triangle  $ABC$  is a right angle and the sides  $BC$ ,  $CA$  and  $AB$  are of lengths  $a$ ,  $b$  and  $c$ , respectively. Each side of the triangle is tangent to the circle  $S_1$  which is of radius  $r$ . Show that  $2r = b + c - a$ .

Each vertex of the triangle lies on the circle  $S_2$ . The ratio of the area of the region between  $S_1$  and the triangle to the area of  $S_2$  is denoted by  $R$ . Show that

$$\pi R = -(\pi - 1)q^2 + 2\pi q - (\pi + 1),$$

where  $q = \frac{b+c}{a}$ . Deduce that

$$R \leq \frac{1}{\pi(\pi - 1)}.$$



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